

**2010**

**Semester II Examination**

**Question/answer booklet**

**MATHEMATICS SPECIALIST**

**3CDMAS**

**Section One**

**CALCULATOR-FREE**

**Time allowed for this section**

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

Question/answer booklet for Section One, containing a removable formula sheet which may also be used for Section Two.

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

**Important note to candidates**

No other items may be taken into the examination room.

It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room.

If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be attempted |  Working time  | Marks available |
| **Section One****Calculator—free** | **8** | **8** | **50 minutes** | **40** |
| Section TwoCalculator—assumed | 12 | 12 | 100 minutes | 80 |
| **Total marks** | 120 |

**Instructions to candidates**

1. Answer the questions in the spaces provided.
2. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.
4. It is recommended that you **do not use pencil** except in diagrams.

1. [1, 1, 5 = 7 marks]

Determine the following integrals.

(a) 

 (b) 

(c)  let 

2. [1, 3, 1 = 5 marks]

(a) Given matrices ***A****,* ***B****,* ***C*** for which ***AB*** *=* ***C***and det ***A*** 0, express ***B*** in terms of ***A***and ***C****.*

(b) Let ***A*** *= *, ***D*** =** and ***C*** = **.

(i) Find the matrix ***DA***

(ii) Find ***B*** if ***AB*** = ***C***.

(c) Find the coordinates of the point of intersection of the planes
*x* + 2*y* + 3*z* = 5, 2*x* – *y* + 2z = 7 and 3*x* – 3*y* + 2*z* = 10.

3. [2 marks]

Determine the equation of the graphs below in polar form.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

B \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. [2, 3 = 5 marks]

Sketch the following regions in the complex plane.

(a) 

(b) 

5. [5 marks]

Find the coordinates of the points where the tangent to the curve  is horizontal.

6. [3 marks]

The plane contains the line . Find *k*.

**7. [2, 2, 1, 2 = 7 marks]**

Let 

(a) Express *z* in the form

(b) Show that 

(c) Express z in polar form.

(d) Find 

8. [6 marks]

The area of the region under the curve  from to  is in between the areas of the two rectangles shown in the diagram.

Show that 

**This page is for extra working.**

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**Question/answer booklet**

**MATHEMATICS SPECIALIST**

**3CDMAS**

**Section Two**

**CALCULATOR-ASSUMED**

**Time allowed for this section**

Reading time before commencing work: 10 minutes

Working time for this section: 80 minutes

**Material required/recommended for this section**

**To be provided by the supervisor**

Question/answer booklet for Section Two. Candidates may use the removable formula sheet from Section One.

**To be provided by the candidate**

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators, CAS, graphic or scientific, which satisfy the conditions set by the Curriculum Council for this course.

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**Structure of this examination**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Number of questions available | Number of questions to be attempted | Working time  | Marks available |
| Section OneCalculator—free | 8 | 8 | 50 minutes | 40 |
| **Section Two****Calculator—assumed** | **12** | **12** | **100 minutes** | **80** |
| Total marks | 120 |

**Instructions to candidates**

1. Answer the questions in the spaces provided.

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3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Correct answers given without supporting reasoning may not be allocated full marks. Incorrect answers given without supporting reasoning cannot be allocated any marks. If you repeat an answer to any question, ensure that you cancel the answers you do not wish to have marked.

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9. [4, 3 = 7 marks]

(a) If *z* is a complex number and find the value of .

(b) Given that *z* = (*b* + i)2 where *b* is real and positive, find the **exact** value of *b* when arg *z* = .

10. [1, 1, 3, 4 = 9 marks]

Given that p = 4i + 2j + 3k, q = 6i + j – 2k and r = *x*i + *y*j + 4k, find:

(a) 2**p** – **q**

(b) a unit vector in the direction of **p**

(c) the value(s) of *x* and *y* if r has length 9 units and is perpendicular to p

(d) Find the exact shortest distance between the point with position vector p and the plane .

11. [4, 3 = 7 marks]

 (a) Find the roots of  in polar form.

(b) For any complex number *z*, prove that **

12. [1, 2, 2 = 5 marks]

In Hollywood, there are three different make up brands that dominate the market: Revlon, Maybelline and Max Factor. People switch from one brand to another all the time.

If they use Revlon this week, there is 0.7 probability they will continue to use it next week, 0.2 probability they will switch to Maybelline, and 0.1 probability they will switch to Max Factor.

If they are now using Maybelline, there is 0.4 probability that they will switch to Revlon, 0.3 probability they will stay with Maybelline, and 0.3 probability they will switch to Max Factor.

If they are now using Max Factor, there is 0.2 probability they will switch to Revlon, 0.3 probability they will switch to Maybelline, and 0.5 probability they will stay with Max Factor.

(a) Express this information as a transition matrix.

(b) If an actress is using Maybelline, what is the probability she will be using Revlon 3 weeks later?

(c) If an actress starts with Maybelline, what is the probability she will be using Max Factor 4 weeks later?

13. [3, 3, 2 = 8 marks]

The following diagram shows a trapezium with vertices A (1, 1), B (4,1), C (4,3), D (2,3). It undergoes a transformation to the new shape (3, 1), (6, 1), (10, 3), (8, 3).

(a) Draw the new shape on the axes above, describe in words the transformation that has taken place and give the appropriate transformation matrix.

(b) Shape  is then transformed by the matrix  to shape .

(i) Draw shape  on the axes above.

(ii) Give a single transformation matrix required to return shape  to trapezium .

(c) State a matrix transformation that will triple the area of the shape.

14. [5, 3 = 8 marks]

The diagram below shows an acute angled triangle ABC. O is the origin with

(a) OA is perpendicular to BC and BO is perpendicular to AC. Use this to prove that .

(b) Prove that CO is perpendicular to AB.

15. [3, 3, 2 = 8 marks]

A life jacket has fallen off the side of a boat in the middle of a lake. The life jacket moves in simple harmonic motion because of the waves. The distance between the lake floor and the life jacket varies between 1.2m and 1.8m and the period of motion is 3 seconds.

At *t* = 0 seconds the life jacket is 1.5m from the bottom of the lake and is moving in a downward direction.

(a) Find an expression for the position of the life jacket, *x*, with respect to the bottom of the lake at any time *t* seconds

(b) Find the maximum velocity of the life jacket and the first time at which this occurs.

(c) Find the total distance the life jacket has travelled in the first 10 seconds.

**16. [7 marks]**

At 9am a spaceship is travelling through the universe with a constant velocity (10**i** - 5**j** + 2**k)** km/h and is at the point with position vector (8**i** + 6**j** - 3**k**) km. At 10am an enemy spaceship flying with constant velocity of (15**i** - 8**j** + 2**k)** km/h is sighted at the point (7**i** - 5**j** + **k**) km. Use the scalar product method to determine the minimum distance between the two spaceships and at what time this occurs.

17. [4, 1 = 5 marks]

The concentration of a drug in the bloodstream is given by the differential equation:

 where *t* is time in seconds.

(a) Find the solution to the differential equation for *C*(*t*) if initially *C* = 0.

(b) After how long will *C* = 0.5?

18. [2, 2, 2 = 6 marks]

A particle moves along the *x*-axis so that its velocity *v* m/s at time *t* (seconds) is given by  . The graph of *v* is shown above for . The position of the particle at time *t* = 0 is 5 m to the right of the origin.

(a) Find the acceleration of the particle at time *t* = 3.

(b) Find the total distance travelled by the particle from time *t* = 0 to *t* = 3.

(c) For  , find the time *t* at which the particle is furthest to the right.

19. [4 marks]

A runner sprints around a circular track of radius 100 metres at a constant speed of 7 m/s, giving an angular velocity rad/s. The runner’s friend is standing at a distance 130 m from the center of the track. How fast is the distance between the two friends changing when the distance between them is 130 m?

20. [6 marks]

Using mathematical induction, prove that the number is divisible by 9 for *n* = 1, 2, ... .

END OF EXAM